Greziako kulturan musikak duen jatorri matematikotik abiatzen da artikulua, noten eta modu eta eskalen sorrera azpimarratuz, beste kultura batzuei buruzko erreferentziak alde batera utzi gabe. Oinarri horren gainean, greziar moduen eta polifoniaren Erdi Aroko garapenari erreparatzen zaio. Arreta berezia eskaintzen zaio musikaren entzute bertikalari, eskala tenperatuari eta harmonia tonalari, musikaren eta arkitekturaren arteko harremanei, ilustratua, hori guztia antzinako adibide modernoekin. Azkenik, musika estokastikoa, fraktalak eta jatorri matematikoko beste musika batzuk ikusten dira. Artikulua etorkizunerako proiekzio batekin amaitzen da.

Giltza-Hitzak: Matematika. Zenbakiak. Musika grekoa. Erdi Aroko musika. Musika estokastikoa. Musikaren entzute bertikala. Eskala tenperatua. Harmonia tonala.

El artículo parte del origen matemático de la música en la cultura griega, subrayando el nacimiento de las notas y de los modos y escalas, sin perder de vista las referencias a otras culturas. Sobre esa base, se atiende al desarrollo medieval de los modos griegos y la polifonía. Se dedica una atención especial a la escucha vertical de la música, la escala temperada y la armonía tonal, las relaciones de la música con la arquitectura, ilustrado, todo ello, con ejemplos antiguos y modernos. Se observa, finalmente, la música estocástica, los fractales y otras músicas de origen matemático. El artículo se cierra con una proyección de futuro.

Palabras Clave: Matemáticas. Números. Música griega. Música medieval. Música estocástica. Escucha vertical de la música. La escala temperada. Armonía tonal.

L'article part de l'origine mathématique de la musique dans la culture grecque, en soulignant la naissance des notes, des modes et des échelles, sans perdre de vue les références à d'autres cultures. Sur cette base, l'attention est portée sur le développement médiéval des modes et de la polyphonie grecs. Une attention particulière est accordée à l'écoute verticale de la musique, à la gamme tempérée et à l'harmonie tonale, ainsi qu'à la relation entre la musique et l'architecture, illustrée par des exemples anciens et modernes. Enfin, il aborde la musique stochastique, les fractales et d'autres musiques d'origine mathématique. L'article se termine par une projection dans l'avenir.

Mots clés: Mathématiques. Nombres. Musique grecque. Musique médiévale. Musique stochastique. Écoute verticale de la musique. Gamme tempérée. Harmonie tonale.

Music and number

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Music theory is intrinsically related to numbers. For example, in the scale, both the Western one and those of other cultures, since they are all based on numerical relationships and proportions. In addition, there are concepts such as rhythm and note duration, which can be represented numerically through musical figures and their values. From another point of view, acoustics is also mathematical. This relationship is essential to understand how sounds are produced and above all to be able to describe it. The frequency of a sound wave determines the pitch of the sound, and this frequency can be expressed numerically in hertz (Hz). Also, concepts such as resonance and interference can be explained mathematically and have a direct impact on the production and perception of sound. Even the musical form ends up having a direct similarity with architecture, although this develops in space and music in time as a function of memory. Perhaps that is why both specialties have always been related and even among the first Greeks, surely because both were based on proportions, music and architecture were considered of divine origin and were placed in the care of Apollo.

The relationship of mathematics with sound, and even more so with artistic sound, is far from being univocal and appears in the most diverse forms throughout time.

To begin with, it would be necessary to determine if mathematics exists by itself in Nature or is a construction of the human mind to have a tool with which to describe that same Nature.

There are those who are convinced that mathematics is something mental that humans have managed to develop to understand the world in which we live. We believe that mathematics resides in physical and biological phenomena, but we have to interpret the formulas that we apply in their results with human criteria so that they end up being a mental construct that people use as a tool of knowledge. Kant himself maintained that it is a product of the human mind and that with them it could not be known what reality itself is like. Currently, a mathematician of the stature of Brian Rotman has stated that mathematics does not exist outside the human mind since it is a cognitive product of it. He has even gone so far as to deny that natural numbers are indeed natural.

On the contrary, representatives of the so-called mathematical realism affirm that, through observation, it is possible to mathematically infer natural phenomena. For example, Roger Penrose maintains that mathematics really exists in Nature and that existence is independent of our observations or the conclusions reached by our mind.

All of the above should be taken into account, but for what interests us here it is not as important to investigate the essence of mathematics as to see what has happened between them and music since, if mathematics exists in Nature is involved in the physical phenomena with which music works, and if it is a construct, music is also a human construction that without man would not exist in Nature. I suppose that there will also be those who think otherwise in this, but it is the position of the person who writes this work.

In classical Greek philosophy there was a real interest in the systematization of the mathematics of music for many reasons, not the least of which was the fact that the harmony of music should reflect the harmony of the cosmos, the stars are governed by proportions and even the distances between planets correspond to musical intervals. All this is something that reaches even Plato, although it is true that this possible music of the cosmos seemed unreal to Aristotle, which did not prevent the entire system of harmony of the spheres from being alive as an idea for several centuries and reaching well into the Baroque with Leibniz, Kepler and Fludd. Today, the concept of music of the spheres has much more to do with a mystical cosmological tradition than with true mathematics, but let us not forget that Pythagoras was the designer of that concept and also, and simultaneously, the one who designed the basic musical theory of West. It is true that the real existence of Pythagoras has also been doubted, although it is believed that he could have existed and that in any case Pythagoreanism is a solid and continuous corpus of doctrine beyond what we know about its alleged author.

Pythagoras (between 570 and 490 BC) believed that celestial bodies, such as the planets, made sounds as they moved through space, and these sounds produced a celestial harmony. This idea suggests a deep relationship between music, mathematics and the structure of the universe. But Pythagoras is also credited with the discovery of the relationship between the lengths of the strings and the sounds they produce. He experimented with strings of different lengths and discovered that string lengths relative to one-half, one-third, and one-fourth of a string produced consonant intervals, such as octaves, fifths, and fourths, respectively. This discovery laid the foundation for the mathematical understanding of musical harmony. Pythagoras also developed the theory of proportional numbers, which he applied to music. According to this theory, musical intervals can be expressed as simple numerical relationships. For example, an octave is a 1:2 ratio, a fifth is 2:3, and a fourth is 3:4.

Although this is the way in which today it is assumed that it was probably the way in which Pythagoras would find the proportions between the notes, for centuries the alleged story of sound hammers was more widespread. According to this legend, when passing by a blacksmith's shop, Pythagoras noticed that three hammers produced harmonious sounds and another did not. Studying them, he came to the conclusion that this was due to the weight ratio that each one of them had. The anecdote was all the rage, especially since it was spread by Nicomanos of Gerasa (1st century AD) and also by Boethius. And the theory was not criticized (and discarded) until Galileo Galilei himself, who indirectly demonstrated that the sound relations of the hammers did not depend on their weight but on their squares, which for those who doubted the very existence of Pythagoras did even. It is hardly credible that he was the author of the famous theorem that bears his name and that some have traced back to the Babylonians or attributed to Euclid like almost all Greek mathematics. But the discovery of the notes through the monochord is real and has an acoustic, physical and mathematical basis whether Pythagoras or another unknown figure discovered it.

In short, Pythagoras, or what he represents, played a crucial role in the history of music by establishing connections between music and mathematics, and by developing theories about the relationship between sounds, numerical proportions, and universal harmony. Not all of his statements are valid today, but the mathematical basis of musical scales does.

His successor was Terpander (5th century BC), to whom the idea of musical modes that provide scales with specific mathematical patterns is attributed. These modes were systems of organization of tones and semitones that provided a structure for musical composition and performance. Each mode had its own emotional and stylistic characteristic. Terpander was a very famous practical musician who definitively defined the Greek lyre, which before him had four strings and he converted it into a seven-stringed instrument. Not without controversy, since he was accused before the Senate of Sparta for it, but it soon prevailed because he was able to escape from the old pentatonic scale and fix the seven-tone diatonic scale that is still in use. For their part, the modes, which were nothing more than the way of arranging the succession of sounds in the scale, were very useful to determine the expression and form of musical pieces. In Greece, since him, seven modes were used that were systematized after Terpander and that influenced later music until the arrival of tonality, although they were adapted, and sometimes misinterpreted, in the Middle Ages. And the Greek modal scale had the peculiarity of being descending while later the Western scales would be understood as ascending, Also Archytas of Taranto (430-360 BC), a convinced Pythagorean through whom most of what we know is known, or at least what was reported, about Pythagoras, solidly intertwined mathematics with musical sounds.

Of course, there was also a certain thought of resistance and so Aristoxenus (354-300 BC), who was a philosopher so important as to have aspired to succeed Aristotle at the head of his school, was an opponent of the Pythagorean theory and he believed that music depended not so much on how the scale was generated as on purely empirical auditory perception. For him, a system was not harmonious because it adjusted to mathematical proportions but because of the effect it had on the ear. He said that the melody was perceived with the senses and was retained by the memory. In any case, his position was not the main one in Greek music because it had philosophical and moral implications that were difficult to ignore, but the idea has continued for centuries as a deaf resistance to the general march of musical art and many times even manifesting itself as main idea around sound art especially in non-specialists who do not understand the mathematical basis so easily and believe that everything is reduced to pleasant or less pleasant combinations of sounds.

What we have just said in fact introduces a distinction that will later become clearer and that is the one between musical sound and what we call noise. In musical sound we can talk about the height of the sound, its intensity and its timbre, the latter linked to the issue of harmonics which will already be talked about. Also other qualities, but these are the basic ones in Greece. The audible sound for humans is between 16 and 20,000 cycles, although these limits may vary depending on the person and their age and, in general, are somewhat narrower. The sounds considered in this way would be pleasant to the ear, although it can be demonstrated that not all analyzable sounds are pleasant depending on the culture or situation. But irregular sounds of pitch that cannot be completely specified and generally complex, which cannot be analyzed, are known as noises and are segregated from the musical. But in the 20th century the noises themselves are already analysable, and the introduction of electronics has made them usable musically. In this way, today we cannot talk about sound versus noise and everything would be reduced to a question of relevance. That is to say, the same sound does not make the same impression, for example if a motorcycle interrupts a summer nap, or if we attend a motorcycling grand prix that would make no sense to take place in silence.

The disadvantage of Greek music in surviving was that its transmission did not occur so much through written notation, but rather was mostly oral, something that was even consistent with Aristotle who considered the book as something rigid that could not cope with verbal richness. In reality, Greek notation is not prior to the 3rd century BC, that is, after the classical period, and is alphabetical and quite primary. In any case, the quasi-divine attribution of its origin to a character whose existence has even been controversial, such as Pythagoras, tells us about the mythical origin of music, something that is insistently repeated in other cultures.

But we should observe that in Greek music the relationship between music and language is very close and this is usually the case with almost all cultures in their beginnings. In this way I think that the debate about whether music comes before language or the other way around loses meaning. Darwin considered music prior to language while others believe the opposite almost violently, as is the case with Skiner. Personally, I think that both things were surely born together, and perhaps also with a gestural element that would include a proto-theater. What is certain is that at the Greek sacred and artistic levels they developed at the same time. Greek poetry was not only verbal, in some way it was sung and this is demonstrable from the oral tradition of the Homeric poems to the lyrics of Sappho. And if today we hear the theater of Sophocles or Aristophanes as an action with text, the Greeks would have been stupefied to see that the chorus spoke because both it and the actors recited with singing intonations. Something that the Renaissance already knew is the attempts to reproduce a theater like the one conceived by the Greeks that makes the totally new genre that will be opera emerge.

In Hindu cosmology, the god Brahma, the creator of the universe, is believed to have been the first musician. Brahma is said to have created music with the help of Saraswati, the goddess of knowledge, arts and music. According to legend, Saraswati played the vina, a stringed instrument, while Brahma recited the Vedas. In this way, music became an essential part of creation and divine expression.

Another mythical story in Hindu tradition involves the god Shiva, one of the main gods of the Hindu trinity. It is said that Shiva is an accomplished musician and dancer, and that he created the cosmic dance (Tandava) to express the creation, destruction and renewal of the universe. His dance is accompanied by divine music, which symbolizes the rhythm and harmony of the cosmos.

These stories would seem to move away from a mathematical conception, but this is not the case because these gods not only create music but also the rules with which men must cultivate it. Swaras are the basic sounds or musical notes in Hindu music. There are seven main swaras: Sa, Re, Ga, Ma, Pa, Dha and Ni. These roughly correspond to the Western notes C, Re, Mi, Fa, Sol, La and Si. In addition to these seven main swaras, there are variant swaras called komal and tivra, which represent alterations in some notes.

Talas are rhythmic patterns in Hindu music. Each tala has a specific time structure that is divided into repetitive cycles of rhythmic units called matras. Talas can vary in complexity, from simple rhythm patterns to highly elaborate, syncopated rhythms. Talas provide a framework for rhythmic improvisation and musical composition.

Indian music uses a relative tuning system rather than an absolute tuning system as used in Western music. This means that the notes do not have fixed reference frequencies, but are defined in relation to the tonic note (Sa) and interval relationships within each specific raga. Raga is the name given to the melodic modes of Indian classical music that, although they have an undeniable improvisatory component, still have broad structural rules to precisely establish the “color” of each raga.

The Greek musical tradition will continue throughout Europe at least until the Renaissance through the thought of Saint Augustine (354-430) who in his musical treatise analyzes the sound phenomenon in mathematical terms to which he adds some moral considerations of a religious nature. Boethius (480-524) also follows these steps and the fact that music during the Middle Ages was part of the teachings of the Quadrivium is more than cited, that is, among the sciences that included, in addition to music, arithmetic, geometry and astronomy, all related to mathematics. All of this with a clear Pythagorean affiliation and that owes much of its expansion to Gerbert of Aurillac (945-1003) who would be Pope under the name of Sylvester II. Further to the East, the Byzantine Empire developed Greek notation with greater success, although it was considered an aid to memory and not an absolute writing. And the Arabs began to introduce an alphabetic writing in a complex modal musical system (maquam) and a scale full of microintervals that makes it very different from the Western one.

Meanwhile, the extensive development of Gregorian chant was done on an oral basis, but a writing was developed from the neumes that little by little would become more varied and exact. It could be said that, at the end of the Middle Ages, the supposedly backward medieval people had a much more perfect musical notation than that of the developed Greeks. And it is almost not necessary, as it is known, to cite the nomenclature of the notes introduced by Guido of Arezzo, who also invented a four-line system for writing that would later develop the staff.

Gregorian chant, which dominates the Middle Ages and is so called because its systematization is attributed to Pope Gregory the Great (540-604). He and his successors compiled the Roman chant from the Christian tradition. Since Guido of Arezzo it has been written in tetragram and its rhythm is prosodic since it is subject to the Latin text. They are sung prayers of a monodic nature, in Latin, and whose form depends on the text and their music is modal using an adaptation of the Greco-Latin modes to what are now called ecclesiastical modes.

Guido created what they called “straight music” or true music, based on a hexachordal system compared to what since Odo of Cluny, around the same time, was “musica ficta” with an improvisatory basis and in which ornamentations were introduced, avoiding, even modifying the notes, the dissonances of the system. Later, “musica riservata” (or secret) would appear, which was not for everyone and which had vast implications regarding the expressiveness of non-religious music, some linked to mathematics that are widely developed in the late medieval or pre-Renaissance motet.

Once an incipient polyphony is launched on the Gregorian monody, it advances and fixes its main form in the motet. The motet is a polyphonic composition with a technique used since the 13th century. Although later the term was applied more broadly, especially to periodic repetition or rhythmic repetition in the tenor voice and other voices in 14th and early 15th century compositions. The tenor is not so much that tessitura of the voice as the original, invented part that cements the polyphony. It is an organization of durations or rhythms called tálea that is repeated throughout a tenor melody whose content or series of pitches (the notes), called color, varied in the number of members of the tálea. Isorrhythmia goes beyond isoperiodicity: not only is the structure of the periods the same, but so are the values of the period scores. The rational organization of the tenor with subdivision into color (notes) and tálea (sound durations, rhythm) was also extended, in this way, to the upper voices. All of this occurs in the so-called Ars Nova, which since the 14th century has been fighting against the so-called Ars Antiqua and which produced not only a musical revolution but even a religious and social controversy in which even a famous bull from a pope intervened, which proclaims in 1322 John XXII trying to prohibit the news of the Ars Nova. The new technique was attributed to Philippe de Vitry (1291-1361) who limited himself to practicing and disseminating it, but did not invent it. Its greatest practitioner is Guillaume de Machaut (1300-1377). The isorhythm motet It constitutes the zenith in terms of rational structuring in the music of the Gothic period. At the same time, the isorhythm creates balance towards the future creation of expressive melody and the increase of harmonic coloration.

From Mannerism onwards, the numerical presence is obscured in the background by the desire to imitate Nature, which is quite novel, although it has another Greek background, which will lead to the theory of the affects that music carries with it from architecture and mathematics towards the field of arts such as literary and representative arts. However, the creation of a vertical listening, which constitutes modern harmony with chords and which is something that does not exist in other cultures where the multiplication of voices is nothing but heterophony and not a system of chord links and relationships. Music, which is an art of time, thus conquers space as painting does through perspective, both systems, perspective and harmony, that develop simultaneously.

Despite everything, the relationship between music and mathematics is not broken, but, on the contrary, in many moments it hardens. A good example has just been given to us with the late medieval world of the isorhythmic motet that applies mathematics to the rhythmic world instead of insisting again on the scalistic.

The mathematical presence is overwhelming in the creation of tonal harmony and in the development of the tempered scale. The tempered scale is a musical tuning system that is based on the mathematical division of musical pitch into equal intervals. Historically, tempered scales have played a crucial role in the evolution of Western music and at the intersection between music and mathematics. In the equal tempered scale, the octave is divided into twelve equal intervals, each of which is approximately one semitone. This system allows modulation between different tones and facilitates the use of all the keys on a keyboard. Adapting this system required careful mathematical calculations and considerations to achieve an equal distribution of sounds within the octave. And even force the results a little to adapt a more irregular sound reality to a system that offered innumerable practical advantages.

The idea of the tempered scale began to gain popularity in Mannerism and the early Baroque, in the 16th and 17th centuries, although ideas about tonal temperance and mathematical divisions of tone developed little by little over centuries, but it was in this period when the equal tempered scale became established as a widely adopted tuned system. The arrival of the tempered scale had important implications for Western music, as it allowed modulation between different tonalities which contributed significantly to the development of tonal harmony and the expansion of the musical repertoire. The equal temperament scale became the basis of the Western tonal system and remains fundamental in contemporary music theory. Its appearance marked a milestone in the evolution of music and its relationship with mathematics, demonstrating the lasting influence of mathematical divisions on music theory.

Equal temperament is the tuning system constructed by dividing the octave into twelve equal parts called tempered semitones. It is also known as a tempered system or tempered scale. Equal temperament is the tuning system most currently used in Western music, and is based on the tempered semitone, equal to the twelfth part of the octave and with a numerical ratio equal to the twelfth root of two, with an intervallic width of 100 cents. The cent is the smallest unit used in measuring sound intervals and is one hundredth of a tempered semitone. The most notable property of equal temperament is the equality of pitch between the enharmonic notes, which is derived from the use of a single type of semitone. Previous tuning systems when moving up the circle of fifths did not achieve coincidences as the intervals produced slightly larger or smaller results in intervals that were not perfect fifths. This difference was called the Pythagorean coma. For the development of tonal harmony the tempered system was absolutely necessary. In fact, it can be said that the introduction of the tempered scale represents a triumph of the mental and practical representation of sound more than of the sound reality itself, which does not entirely adjust to it but ends up doing so based on a special education of the ear that forces nature for the benefit of theoretical logic.

An always latent problem with respect to the organization of the notes was that of finding a reference point from which they would all be tuned. Obviously, this, at first, could only be found at the local level but it was increasingly urgent to find a general rule applicable everywhere, which was neither easy nor early. Until the 16th century, the tuning standard could vary not only in different places but also on instruments in the same city. A high frequency tuning could damage the instrumental strings, which were usually made of gut and were therefore fragile, and also the organ pipes and was equally a torment for the singers.

From what we have just said, it is often stated that the tuning in the Renaissance and Baroque was lower than today. That is not true and depends entirely on places and times, because there are examples of historical organs tuned higher than the current standard and already in the 1500s Michael Praetorius (1571-1621) was clamoring for 480 Hz tunings, notably higher than the modern, and therefore more tense than the current ones. In the 18th century, depending on the location, there are examples between 380 Hz and

480 Hz (the latter applied to the organs used by Bach in Leipzig) and the appearance of hairpin tuning forks allowed a certain standardization, at least in genres such as opera that included it. is going down. In the 19th century, the orchestral tuning tended to be raised, especially when the gut strings were changed to metallic ones. In 1859 the French government established by law that the central "A" (referent to the tuning of the rest of the notes) should be 435 Hz. In 1936 an international conference recommended that this "A" be 440 Hz, which was adopted in 1955 by the World Organization for Standardization as the ISO 16 standard. This is theoretically done all over the world, although there are orchestras that go up three or four hertz and in early music tunings up to a semitone lower are usually used.

Tonal harmony, which dominated from the 17th to the 20th centuries, and still remains in many artistically undemanding musical genres, is based on the mathematical relationships of sounds to create a coherent and pleasant musical structure. The mathematical understanding of proportions and musical interval relationships was fundamental to the development of tonal harmony. The systematization of chords and harmonic progressions is based on proportional relationships between the notes. Musical intervals, such as fifths, fourths, and thirds, have precise mathematical dimensions that influence how they combine and sound together.

The theory of tonal resolution in Western harmony is rooted in mathematical principles of tension and relaxation, creating a directional and balanced feel to the musical work. Tonal harmony, being a perfectly organized and structured system, has deep mathematical foundations that influence the way musical works are composed, performed and perceived. This concatenation between music theory and mathematics has been the true basis of musical composition for a long time. When Rameau systematizes, which he does not invent in any way, tonal harmony, what he is doing is establishing a musical nomenclature for phenomena that could have been expressed in mathematical language.

One of the great problems of music in its notes and scales is that it must take into account the issue of harmonics since each note is not only itself. It is not a pure sound but accompanied by others that are not less important because they are fainter, since the timbres and their differences are derived from their arrangement and greater or lesser presence.

The fundamental sound of a note is the frequency whose pitch is perceived. But everyone knows that an "a" on a flute does not sound the same to anyone as an "a" on a clarinet. The sound is perceived to be the same, it has the same vibrations, but its "color" is not the same at all. For centuries, timbre was not very important and harmonics were perhaps little studied, but modernly it is almost the fundamental basis of composition. The same note on various instruments changes color depending on how the harmonics are arranged. And a pure note, without harmonics, actually practically does not exist in nature and is only achieved modernly with electronic devices. In a given sound, up to 16 harmonic sounds can be studied modernly, which, depending on their order, have their function. And not all of them are really good-sounding, but they are comparable to standard tuning. But it is not uncommon to find sounds that have many more harmonics up to 30 or 40, but, starting at 16, the distances between them are less than a semitone, which introduces us to the complex world of microtones, so used today. The totality of harmonic sounds present in a some modern musical trend.

On the other hand, timbre is a complex and quite changing concept since its spectrum must be taken into account, which indicates its energy according to the distribution of harmonics. There is also the variation in amplitude over time that constitutes the dynamic envelope as well as the intensity, which is the energy concentration of the frequency. A sound wave has to distinguish its attack, which is the time in which it reaches its maximum amplitude, the decay, which is the time in which the wave falls from its maximum to a stationary state, the time in which it remains in that state, and the release. that goes from the steady state to the disappearance of the sound.

In the avant-garde music of the 20th century, mathematics becomes more explicit, which does not mean that it is used more than before, but that it is talked about more openly. The appearance of dodecaphonism and its extension in serialism has a mathematical basis that emerges very directly and that even has essentially numerical material like many of the things we have just seen. But what stands out most in modernity is not so much the thread of number that has led us to it since Pythagoras but the theme of proportions that inserts us into geometry and spatiality. Nor is it something new because it has been present in some way since ancient times, in many cases explicitly and others in a subconscious way that clings to musical techniques and is reflected in them. Even musical sound is not alien to the place of its origin or to the variation and distance of the listener. The Greeks knew this, always attentive to the relationship between music and architecture when it came to achieving tetaros with an astonishing acoustic capacity. But the medievals also knew it when polychoral music emerged for certain churches and we see how the musicians practicing in San Marcos in Venice used it in a masterful and acoustically surprising way.

But if we want to relate the development of music with other artistic disciplines we must resort to the field of form and consider the problem of proportions therein. This happens with all the arts, although architecture has been mentioned more since ancient times since phenomena such as the golden ratio, the Fibonacci series, etc. are more recurrent in it. Speaking of proportions, an element that appears repeatedly throughout History is that of the golden ratio, or divine proportion, which has dazzled musicians over time, but also architects and mathematicians and which has not been foreign to natural philosophy since it is observed, or believed to be observed, that it is present in geology, botany and many other parts of the natural sciences.

The golden ratio appears when the quotient between the sum of two quantities and the largest number is equal to the quotient between the largest number and the smallest number. It is represented by the Greek letter phi (φ) and has an approximate value of 1.618. Mathematically, the golden ratio "phi (φ)", results from $(a + b) / a = a / b = \varphi$. It has been used in different arts at various times and in music it is found repeatedly in baroque authors and from other times. It has been found in many works by Johann Sebastian Bach and other baroque authors and in reality it is not necessary for the author to know it expressly, but rather it is intuitively present in the musical forms and in the creative sensibility of the masters involved.

Another proportional method universally used in the arts is the so-called Fibonacci series. It was expressed by the mathematician Leonardo of Pisa (12th-13th centuries), sometimes called Fibonacci. The Fibonacci series, a mathematical sequence in which each number is the sum of the two previous ones (0, 1, 1, 2, 3, 5, 8, 13, etc.), was used by its author for something as unartistic as rabbit breeding, but it has been used profusely (along with the golden ratio) by painters, sculptors, architects, designers and, of course, musicians. This series has been used by some modern composers interested in Hindu culture along with the Narayana series.

Narayan Pandit (14th century) was a Hindu mathematician who proposed a series similar to that of Fibonacci. This series, instead of adding the two previous numbers to obtain the next one, adds the previous number with the number that was two places before. The Narayana series begins with 0, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, etc. Instead of rabbits, like Fibinacci, Narayana applied it to cows and some modern composers have used it. Probably the first was the American Tom Johnson in a piece called *The Cows of Narayana*.

Express relationships between music and architecture through mathematics can take us far from current times and there are surprising examples. On March 25, 1436, Pope Eugene IV, taking refuge in Florence during the period of his confrontation with the Council of Basel, solemnly consecrated *Santa Maria dei Fiore*, the city's cathedral, which debuted the impressive dome designed by Filippo Brunelleschi. For the occasion, the motet *Nuper rosarum flores* was sung, composed by the Franco-Flemish Guillaume Dufay (1397-1474), who had worked in Bologna before entering Rome in the service of the Papal Chapel of Eugene IV. The motet uses the same mathematical proportions as the cathedral dome, as noted by musicologist Charles Warren, modeling the contrapuntal way of treating the voices of the tenor parts I and II.

An example as brilliant and even more obvious than that of Dufay-Brunelleschi, and also made by a single artist who was both an architect and a musician, is that of Iannis Xenakis who used the same structural approach for a piece of music and a building.

Xenakis's intimate relationship with mathematics is very broad and addresses various perspectives; Let us think about the spatialization of its various polytopes or its use of the various aspects of stochastic music. He assumes his dual nature as composer and architect in the score of *Metastaseis* and in the plans for the Philips Pavilion for the 1958 Brussels World's Fair. For the World's Fair, architect Le Corbusier was commissioned to create a pavilion for the Philips company that It would serve to host a show. Given his workload, he referred the design to Iannis Xenakis, who was then collaborating as an architect in his studio. He conceived an asymmetrical building formed by nine hyperbolic paraboloids that was based on the same principles as a previous musical composition of his: *Metastaseis*. Le Corbusier called the building *Electronic Poem* and asked Edgard Varèse to create a work with that title, which Xenakis himself complemented another electronic work, *Concret PH*. Although the building was dismantled at the end of the Expo, its plans and abundant documentation remain.

The twin musical work, *Metastaseis*, was composed by Xenakis in 1954 and premiered at the Donaueschingen Festival on October 16, 1955 under the direction of Hans Rosbaud. It is designed for 65 instruments, of which 12 are wind, 7 are percussion and 46 are string, which are the ones that lead the composition, starting from a unison on the G note and then opening into individual parts. Xenakis used models extracted from hyperbolic paraboloids and also from Modulor, a measurement system introduced by Le Corbusier based on human proportions, the golden ratio and the Fibonacci series. Shortly after, he applied the principles together with Le Corbusier in another architectural work, such as the Tourette convent.

Xenakis was always interested in mathematics and based his musical compositions on it. And he always made clear his dual status as musician and architect, which, when translated into writing, was reflected in his book Musique Architecture published in 1976. He introduced the concept of “stochastic music” or music based on random processes. Instead of following a previously defined musical structure, such as musical forms in general, he incorporates elements of randomness or probability into the creation. In this way he used statistical procedures, probability laws, the theory of games or even the expansion of gases according to the Maxwell-Boltzmann theory.

The fact that Xenakis introduced stochastics has to do with the exhaustion that he observed in twelve-tone music and, in general, in serial processes, which did not endear him to the serial avant-garde and it is known that he never got along well with Boulez, despite because he had a good mathematical background.

Xenakis frequently resorted to “Markov chains” whereby the probability of an event occurring depends only on the previous event. He used randomly generated numbers to design his music and often used graphics to describe very complex sound realities. And we must deny the reproach that was often made against him that the mathematical procedure was important in him, but not the sound result it offers. It isn't true. That sound result that must be heard is what he wants and it is through these procedures that it can be obtained.

Xenakis lives in a context in which electroacoustic music already exists, which comes to rescue the so-called noise as an essentially musical matter. Although preceded by previous studies and devices, it actually emerged just after the Second World War in two different focuses. The first in Paris around French state radio broadcasting where Pierre Schaeffer and Pierre Henry develop music that starts from the recording of any noise and its conversion into a musical work using a montage and filtering system. This is what was called “concrete music.” At the same time, on the radio in Cologne, with Herbert Eimert and Karlheinz Stockhausen, another more apparently scientific branch is developed in which an electronic sound is created through generators to work with. At the same time, in the United States, a similar process is developing around Columbia-Princeton University. All of them will come to an integration in what has since been known as electroacoustic music that is practiced in complex studios until the appearance of the synthesizer that spread it in all musical fields, including the most commercial ones. Xenakis, of course, practiced his own electroacoustic work and today there is no composer who cannot resort to it at some point so that everything that is talked about music at least in the last seventy-five years has to encompass the instrumental, the vocal and the electroacoustic.

It is not necessary to affirm that Xenakis is the best known of the modern composers who are explicitly mathematical in his creative approach. He may even be the greatest in that he is the initiator, but he is by no means the only one. So much so that, many years later and at the point where we are now in the 21st century, there would hardly be composers of a certain magnitude who did not often resort to mathematics in a totally normal way. And not even the cultivators of the new complexity with hispid stochastic processes, even the apparently simple minimalists of all kinds, of which there are, escape from that.

As an example we could add how common it is today in music (and I think also in art) to see how composers resort to fractals. What is meant by a fractal is a geometric object that has a fundamentally irregular structure that is repeated at any scale considered. It is a type of work that has possibly always been done in some way, even if it was not called that, and in plastic art, oriental tapestries or the ornamentation of Islamic palaces are cited as key examples.

“Modern” fractals are expanded through the work of the mathematician Benoit Mandelbrot who in the seventies of the 20th century developed a series of studies on a particular type of fractals. His impact was enormous on the plastic arts but the musicians were not left behind in the process. So much so that fractal procedures have been studied in Bach and other composers of the past, preferably Baroque, but also in others from the 20th century such as Bartók or Messaien. At this point it could be said that all composers use them or have used them at some point.

A very influential post-Xenakis example of other mathematical tendencies in music is given by the so-called spectral composition or spectralism. Its origin is French and occurred in the seventies around the group L'tineraire and authors such as Gérard Grisey, Tristan Murail, Michel Levinas, Hughes Dufourt and others, both theorists and practitioners of this type of technique. It is based on the spectral analysis of each note which, as is known, not only carries the main note but a series of harmonics that can be studied to infinity, perhaps not entirely acoustically but physically and mathematically. The traditional importance of harmonics lies in the problems of timbre or “color” of sounds but, when harmonics move away, they submerge in a microtonal universe, naively not expected if we think about the true mental construct, more than sound, which is the tempered scale.

We have said that it is an initially French movement, but that it has had an influence on musicians from other latitudes, whether or not they followed it in France itself. Some British, like Jonathan Harvey, took it as a kind of promised land and others used it as another possibility, such as Georges Benjamn, also British, the Finnish Katja Saariaho or the Spanish José Manuel López López. And for many others it has become a work tool like any other.

Everything we have said affects the scientific nature of music, which does not imply that it is not also an art. Someone as important in these territories as René Thom, whose theory of catastrophes was also used by Xenakis, stated that all science is the study of phenomenology. And we will add that there are few things as phenomenological as music. Music and performance have gone together since their origins and this is almost an anthropological universal, even though in many cultures it is disguised behind the sacred. And it does not seem that they will be able to separate themselves in the future.

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